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THE USE OF TRIANGULAR FUZZY NUMBERS IN FUZZY ANALYSIS OF CRITICAL PATHS IN PROJECT PLANNING

BY

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Abstract. In order to plan and to schedule projects of large size there have been introduced the Critical Path Method (CPM) techniques. With large consent among specialists as being a valuable tool, they are deterministic with regard to the duration assigned to perform the activities and with the results generated. Since the actions are executed under uncertain terms and conditions, the entire chain and outcome has a certain level of uncertainty. CPM network calculations can be performed by fuzzy operations and the activities durations can be represented by fuzzy numbers. The paper presents an analytical method for measuring the criticality in a fuzzy project network, where a triangular fuzzy number represents the duration time of each activity. There it is based a method using the ranking value of a fuzzy number. To find the critical path, there is used and applied a defuzzification formula for triangular fuzzy number to the total slack for each activity in the fuzzy project network. Then, there is revealed the comparison of the results: the result got by defuzzification formula for triangular fuzzy number against the result got by the present method. The comparison proves that the method described in this paper can be successfully applied with triangular fuzzy numbers.

Keywords: fuzzy project network; triangular fuzzy numbers; ranking method; fuzzy total slack; defuzzification.

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1. Introduction

Project implementation may be challenging for unexperienced project managers. Critical path method (CPM) has been proved to be a useful tool to overcome the hurdles in managing projects when the task times are deterministic and known. To reduce the project duration, the aim of CPM is to identify critical activities on the critical path so that resources may be focused on these activities. A determined time duration of each activity triggers a successful implementation of CPM. However, in practice, when some of the activities are performed for the first time, this requirement is not so easy to fulfill in practice.

Program Evaluation and Review Technique (PERT) based on probability theory has been applied. PERT estimated the activity duration using three times scenarios: optimist, pessimist and the most likely. Over the time this approach has been subject of debate among researchers.

In 1965, Zadeh (Zadeh, 1965) introduced the concept of fuzzy set, designed to overcome this kind issues. This way the fuzzy theory has been advanced as a relevant tool for dealing with imprecise activity durations.

Some authors have analyzed the pragmatism of the fuzzy theory for critical path analysis.

Dubois and Prade (Dubois and Prade, 1988) and Hapke and Slowinski (Hapke and Slowinski, 1996) used fuzzy arithmetic operations to compute the latest respective the earliest starting time of each activity of a project.

In order to find out possible critical paths in a fuzzy project, Chen *et al*. (Chen and Chang, 2001) used the defuzzification method.

In fuzzy environment, the ranking technique is applied for the fuzzy number comparison. For its implementation it is utilized a useful ranking method (Liang and Han, 2004). In this paper, defuzzification of triangular fuzzy numbers is made with the formula presented in (Shankar *et al*., 2010).

Grounded on the ranking value method of a fuzzy number, the paper gives a solution to the fuzzy critical path problem with triangular representation of the activity duration and presents the solving of a numerical example.

2. Fuzzy Concept

A fuzzy set can be mathematically be built by assigning a value representing its grade of membership in the fuzzy set to each and every possible individual in the universe of discourse (Zadeh, 1965). The fuzzy number A is a fuzzy set whose membership function $\mu_A(x)$ complies with the following cumulative conditions (Shankar *et al*., 2010): $\mu_A(x)$ is piecewise continuous, $\mu_A(x)$ is a convex fuzzy subset and $\mu_A(x)$ is the normality of a fuzzy subset.

A triangular fuzzy number A can be defined by a triplet (a, m, b) whose membership function is:

$$\mu_A(x) = \begin{cases} \frac{x-a}{m-a}, a \le x < m \\ 1 \quad , x = m \\ \frac{b-x}{b-m}, m < x \le b \\ 0 \quad , otherwise \end{cases} \quad \text{where } a, m, b \in \mathbb{R} \qquad (1)$$

Let consider $A_i = (a_i, m_i, b_i), i = \overline{1, n}$ to be fuzzy numbers with membership functions μ_{Ai} respectively.

Let $x_1 = min\{a_1, a_2, \dots, a_n\}$ and $x_2 = max\{b_1, b_2, \dots, b_n\}$.

The ranking value of the fuzzy number A_i is defined as below:

$$R(A_i) = \beta \left[\frac{b_i - x_1}{x_2 - x_1 + b_i - m_i}\right] + (1 - \beta)\left[\frac{x_2 - a_i}{x_2 - x_1 + m_i - a_i}\right] \qquad (2)$$

where β can be calculated by:

$$\beta = \left[\sum_{\substack{i \\ A_{ij} \in ACT}} \sum_j \frac{b_{ij} - a_{ij}}{(m_{ij} - a_{ij}) + (b_{ij} - m_{ij})}\right] / t \qquad (3)$$

β is called the decision maker's risk attitude index. The ACT represents the set of all activities and $t$is the total number of activities in the project network.

For a fuzzy number $A_i, i = \overline{1, n}$ we define:

$$m_i = min\{x/\mu_{Ai}(x) = 1\} + max\{x/\mu_{Ai}(x) = 1\} \qquad (4)$$

where μ_{Ai} are the membership functions of A_i.

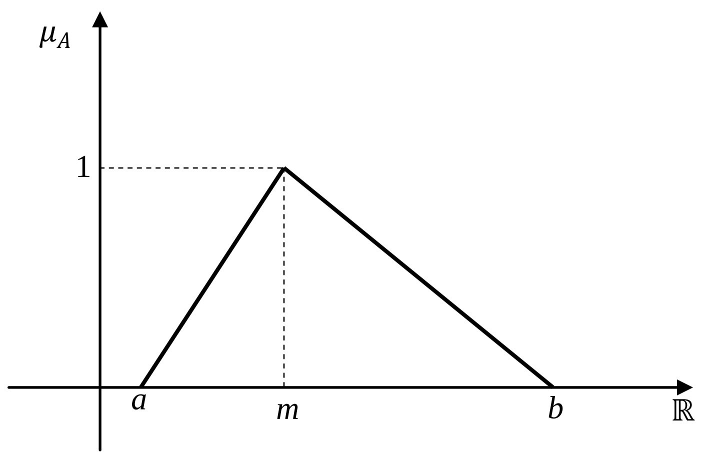

Fig. 1 – A triangular fuzzy number.

The ranking process of the fuzzy numbers A_i and A_j is performed based on the below criteria:

$$A_i > A_j \Leftrightarrow R(A_i) < R(A_j), or\ R(A_i) = R(A_j) \quad (5)$$

$$A_i = A_j \Leftrightarrow R(A_i) = R(A_j) \quad (6)$$

Let $A = (a_1, m_1, b_1)$ and $B = (a_2, m_2, b_2)$ be two triangular fuzzy numbers. Then, the maximum operation between A and B is denoted by:

$$\max(A, B) = (\max(a_1, a_2), \max(m_1, m_2), \max(b_1, b_2)) = (a, m, b) \quad (7)$$

The fuzzy activity duration of activity A_{ij} within a project network, named as FET_{ij} , is represented by triangular fuzzy number $FET_{ij} = (a_{ij}, m_{ij}, b_{ij})$ where a_{ij}, m_{ij}, b_{ij} are minimum, moderate and maximum values of assessing activity duration for A_{ij}. The decision-maker can evaluate stand alone, based on his subjective judgement, knowledge and skills, the fuzzy activity duration $FET_{ij} = (a_{ij}, m_{ij}, b_{ij})$ only after all information available regarding the activity A_{ij} is in place.

The extended algebraic operations of any of the two fuzzy activity duration $FET_1 = (a_1, m_1, b_1)$ and $FET_2 = (a_2, m_2, b_2)$ can be expressed as:

Addition $\oplus$:

$$FET_1 \oplus FET_2 = (a_1, m_1, b_1) \oplus (a_2, m_2, b_2) = \\ = (a_1 + a_2, m_1 + m_2, b_1 + b_2) \quad (8)$$

Subtraction $\ominus$

$$FET_1 \ominus FET_2 = (a_1, m_1, b_1) \ominus (a_2, m_2, b_2) = \\ = (a_1 - b_2, m_1 - m_2, b_1 - a_2) \quad (9)$$

Defuzzification of fuzzy set is used to obtained crisp values which represent uncertain data. Since it is easier to make a comparison between two or more crisp values, the decision-making process is improved by turning a fuzzy number into a crisp number. Various defuzzification methods are implemented for different types of fuzzy numbers. By using the expression (Shankar *et al.*, 2010) for a triangular fuzzy number $A = (a, m, b)$, we can get the crisp value representing the fuzzy number:

$$Centroid(A) = \frac{a+m+b}{3} \quad (9)$$

A critical activity can be defined as the one whose total slack is equal to zero and we will use above formula to defuzzification the fuzzy total slack of each activity.

3. Numerical Example

In order to prove the method described in the (Rusu, 2017) let's consider a hypothetical numerical example.

To develop a new portfolio of goods, a company has to run a relevant project that contains six different activities presented in Fig. 2. The duration of the fuzzy activity of each action is presented in Table 1. The target is to find the fuzzy critical path.

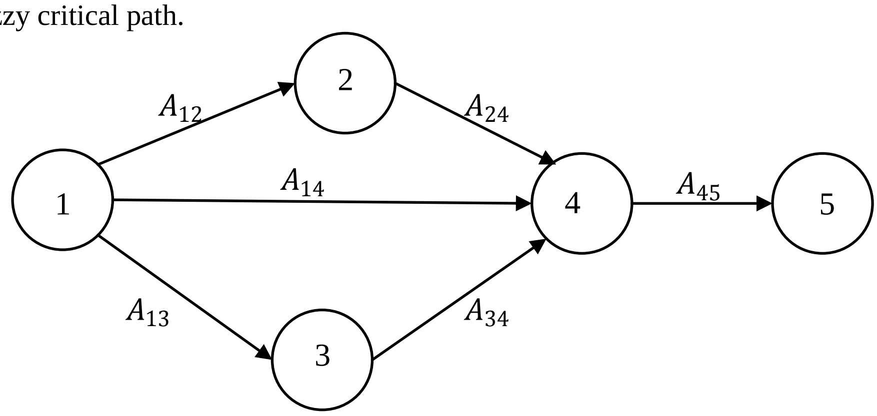

Fig. 2 – A project network.

Table 1
Fuzzy Activity Time for Each Activity

Nr.	Activity Code	Activities directly preceding	Fuzzy activity duration (weeks)
1	A_{12}	-	(3,5,7) *approximately 5 weeks*
2	A_{13}	-	(5,10,15) *approximately 10 weeks*
3	A_{14}	-	(1,3,5) *approximately 3 weeks*
4	A_{24}	A_{12}	(2,4,6) *approximately 4 weeks*
5	A_{34}	A_{13}	(1,2,3) *approximately 2 weeks*
6	A_{45}	A_{14}, A_{24}, A_{34}	(6,8,10) *approximately 8 weeks*

A MATLAB computer-based program is developed to implement the above presented methodology. It is designed to run fuzzy arithmetic calculations for the project network.

The outcome of the analysis of the fuzzy critical path is shown in Table 2, together with the fuzzy total slack.

Table 2

Values of FET_{ij}, FES_{ij}, FEF_{ij}, FLS_{ij} and FLF_{ij} and Fuzzy Total Slack of Each Activity

Act. Code	Duration (weeks)	Fuzzy Earliest Start	Fuzzy Earliest Finish	Fuzzy Latest Start	Fuzzy Latest Finish	Fuzzy total slack
A_{12}	$(3,5,7)$	$(0,0,0)$	$(3,5,7)$	$(-11,3,17)$	$(-4,8,20)$	$(-11,3,17)$
A_{13}	$(5,10,15)$	$(0,0,0)$	$(5,10,15)$	$(-16,0,16)$	$(-1,10,21)$	$(-16,0,16)$
A_{14}	$(1,3,5)$	$(0,0,0)$	$(1,3,5)$	$(-3,9,21)$	$(2,12,22)$	$(-3,9,21)$
A_{24}	$(2,4,6)$	$(3,5,7)$	$(5,9,13)$	$(-4,8,20)$	$(2,12,22)$	$(-11,3,17)$
A_{34}	$(1,2,3)$	$(5,10,15)$	$(6,12,18)$	$(-1,10,21)$	$(2,12,22)$	$(-16,0,16)$
A_{45}	$(6,8,10)$	$(6,12,18)$	$(12,20,28)$	$(2,12,22)$	$(12,20,28)$	$(-16,0,16)$

All possible paths are $P = \{(1,2,4,5), (1,4,5), (1,3,4,5)\}$. The fuzzy completion duration and the ranking value of fuzzy completion duration is calculated for all paths. The results are presented in Table 3.

Table 3

Fuzzy Activity Time for Each Path

Nr.	Paths	Fuzzy completion time of path P_i $FCPM(P_i)$	Ranking Value of $FCPM(P_i)$
1	$P_1 = (1,2,4,5)$	$FCPM(P_1) = (-38,6,50)$	$R(FCPM(P_1)) = 0{,}690$
2	$P_2 = (1,4,5)$	$FCPM(P_2) = (-19,9,37)$	$R(FCPM(P_2)) = 0{,}674$
3	$P_3 = (1,3,4,5)$	$FCPM(P_3) = (-48,0,48)$	$R(FCPM(P_3)) = 0{,}657$

Since $R(FCPM(P_1)) > R(FCPM(P_2)) > R(FCPM(P_3))$, the fuzzy critical path is P_3. Hence, the project completion duration is approximately 20 weeks, *i.e.* $(12,20,28)$.

Next step, by using formula (9), it is to defuzzification the total float of each activity. The results are shown in Table 4.

Table 4

Defuzzified Values of Total Slack Time for Each Activity

Act. Code	Duration (weeks)	Fuzzy total slack	Defuzzified value
A_{12}	$(3,5,7)$	$(-11,3,17)$	3
A_{13}	$(5,10,15)$	$(-16,0,16)$	0
A_{14}	$(1,3,5)$	$(-3,9,21)$	9
A_{24}	$(2,4,6)$	$(-11,3,17)$	3
A_{34}	$(1,2,3)$	$(-16,0,16)$	0
A_{45}	$(6,8,10)$	$(-16,0,16)$	0

Because the crisp slacks of activities A_{13}, A_{34}, A_{45} are equal to 0 the conclusion is that those activities are critical and the critical path is $P_3 = (1,3,4,5)$. This result complies with the result obtained on the ranking value of a fuzzy number.

4. Conclusions

There have been computed fuzzy completion duration for each path in a project network to determine the critical path. Using the method based on ranking value of triangular fuzzy numbers, a numerical example was solved. In the end, there was performed the comparison of the results: the ones got by defuzzification formula for triangular fuzzy numbers and the others obtained by ranking method.

REFERENCES

Chen S.M., Chang T.H., *Finding Multiple Possible Critical Paths using Fuzzy PERT*, IEEE Transactions on Systems, Man and Cybernetics, Part B, Cybernetics, **31**, 6 (2001).

Dubois D., Prade H., *Possibility Theory: An Approach to Computerized Processing of Uncertainty*, Plenum Press, New York, 1988.

Hapke M., Slowinski R., *Fuzzy Priority Heuristics for Project Scheduling*, Fuzzy Sets and Systems, **83**, *3*, 291-299 (1996).

Liang G.S., Han T.C., *Critical Path Analysis Based on Fuzzy Concept*, International Journal of Information and Management Sciences, **15**, *4*, 29-40 (2004).

Rusu A., *A Fuzzy Approach to Critical Path Method in Projects Planning*, Bul. Inst. Polit. Iaşi, **63 (67)**, *4*, 51-59 (2017).

Shankar N.R., Sireesha V., Rao P.B., *An Analytical Method for Finding Critical Path in a Fuzzy Project Network*, Int. J. Contemp. Math. Sciences, **5**, *20*, 953-962 (2010).

Zadeh L.A., *Fuzzy Sets*, Information and Control, **8**, *3*, 338-353, online at https://doi.org/10.1016/S0019-9958(65)90241-X, (1965).

UTILIZAREA NUMERELOR FUZZY TRIUNGHIULARE ÎN ANALIZA FUZZY A METODEI DRUMULUI CRITIC ÎN PLANIFICAREA PROIECTELOR

(Rezumat)

Pentru a planifica și a programa proiecte de anvergură au fost dezvoltate tehnicile drumului critic (CPM). Exista un consens între specialiști în a aprecia pozitiv tehnicile drumului critic deși ele sunt deterministe în ceea ce privește durata atribuită activităților și rezultatele obținute. Atât timp cât acțiunile sunt executate sub condiții și termene incerte, întregul lanț și rezultatul final în sine au un anumit grad de incertitudine. În determinarea drumului critic, calculele pot fi efectuate prin operații fuzzy şi duratele activităților proiectului pot fi reprezentate prin numere fuzzy. Lucrarea prezintă o metodă analitică pentru măsurarea criticității într-o rețea de proiecte fuzzy, unde durata fiecărei activități este exprimată printr-un număr fuzzy triunghiular. Pentru a găsi drumul critic se utilizează și se aplică o formulă de defuzificare pentru rezerva

totală exprimată printr-un număr fuzzy triunghiular pentru fiecare activitate din rețea. În final se compară rezultatele: rezultatul obținut prin formula de defuzificare pentru numărul fuzzy triunghiular în raport cu rezultatul obținut prin prezenta metodă. Comparația demonstrează că metoda descrisă în această lucrare poate fi aplicată cu succes cu numere fuzzy triunghiulare.